## Nepriame meranie - príklad 7 – maticový prístup

Vypočítajte obsah lichobežníka z nameraných hodnôt jednotlivých strán *a*, *c*, *v* a určite neistotu merania. Výsledok vyjadrite s pravdepodobnosťou 95 %.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***n*** | ***a*** | ***b*** | ***v*** | $$c$$$$(a\_{i}-\overbar{a})$$ | $$d$$$$(b\_{i}-\overbar{b})$$ | $$e$$$$(v\_{i}-\overbar{v})$$ | $$c^{2}$$ | $$d^{2}$$ | $$e^{2}$$ | $$c∙d$$ | $$c∙e$$ | $$d∙e$$ |
| 1 | 33,75 | 65,70 | 15,50 | 0,016 | 0,094 | 0,001 | 2,56E-04 | 8,84E-03 | 1,00E-06 | 1,50E-03 | 1,60E-05 | 9,40E-05 |
| 2 | 33,70 | 65,40 | 15,48 | -0,034 | -0,206 | -0,019 | 1,16E-03 | 4,24E-02 | 3,61E-04 | 7,00E-03 | 6,46E-04 | 3,91E-03 |
| 3 | 33,60 | 65,65 | 15,49 | -0,134 | 0,044 | -0,009 | 1,80E-02 | 1,94E-03 | 8,10E-05 | -5,90E-03 | 1,21E-03 | -3,96E-04 |
| 4 | 33,75 | 65,60 | 15,52 | 0,016 | -0,006 | 0,021 | 2,56E-04 | 3,60E-05 | 4,41E-04 | -9,60E-05 | 3,36E-04 | -1,26E-04 |
| 5 | 33,74 | 65,65 | 15,51 | 0,006 | 0,044 | 0,011 | 3,60E-05 | 1,94E-03 | 1,21E-04 | 2,64E-04 | 6,60E-05 | 4,84E-04 |
| 6 | 33,80 | 65,68 | 15,55 | 0,066 | 0,074 | 0,051 | 4,36E-03 | 5,48E-03 | 2,60E-03 | 4,88E-03 | 3,37E-03 | 3,77E-03 |
| 7 | 33,75 | 65,72 | 15,45 | 0,016 | 0,114 | -0,049 | 2,56E-04 | 1,30E-02 | 2,40E-03 | 1,82E-03 | -7,84E-04 | -5,59E-03 |
| 8 | 33,80 | 65,50 | 15,50 | 0,066 | -0,106 | 0,001 | 4,36E-03 | 1,12E-02 | 1,00E-06 | -7,00E-03 | 6,60E-05 | -1,06E-04 |
| 9 | 33,75 | 65,58 | 15,51 | 0,016 | -0,026 | 0,011 | 2,56E-04 | 6,76E-04 | 1,21E-04 | -4,16E-04 | 1,76E-04 | -2,86E-04 |
| 10 | 33,70 | 65,58 | 15,48 | -0,034 | -0,026 | -0,019 | 1,16E-03 | 6,76E-04 | 3,61E-04 | 8,84E-04 | 6,46E-04 | 4,94E-04 |

$$S=\frac{a+b}{2}∙v=\frac{\overbar{a}+\overbar{b}}{2}∙\overbar{v}=\frac{33,734+65,606}{2}∙15,499=769,83533 mm^{2}$$

$$u\_{A}^{2}=A\_{x}∙U\_{x}∙A\_{x}^{T}$$

$$A\_{x}=\left(\frac{∂S}{∂a};\frac{∂S}{∂b};\frac{∂S}{∂v}\right) ⟹ \left(\frac{v}{2};\frac{v}{2};\frac{a+b}{2}\right)$$

$$u\_{cc}=\frac{1}{n∙(n-1)}\sum\_{}^{}\left(x\_{i}-\overbar{x}\right)^{2}=\frac{1}{10∙(10-1)}\sum\_{}^{}\left(a\_{i}-\overbar{a}\right)^{2}=\frac{1}{90}\sum\_{}^{}\left(a\_{i}-33,734\right)^{2}=\frac{0,03004}{90}=3,338∙10^{-4}$$

$$u\_{dd}=\frac{1}{n∙(n-1)}\sum\_{}^{}\left(x\_{i}-\overbar{x}\right)^{2}=\frac{1}{10∙(10-1)}\sum\_{}^{}\left(b\_{i}-\overbar{b}\right)^{2}=\frac{1}{90}\sum\_{}^{}\left(b\_{i}-65,606\right)^{2}=\frac{0,08624}{90}=9,582∙10^{-4}$$

$$u\_{ee}=\frac{1}{n∙(n-1)}\sum\_{}^{}\left(x\_{i}-\overbar{x}\right)^{2}=\frac{1}{10∙(10-1)}\sum\_{}^{}\left(v\_{i}-\overbar{v}\right)^{2}=\frac{1}{90}\sum\_{}^{}\left(v\_{i}-15,499\right)^{2}=\frac{0,0649}{90}=7,211∙10^{-5}$$

$$u\_{cd}=u\_{dc}=\frac{1}{n∙(n-1)}\sum\_{}^{}\left(x\_{i\_{k}}-\overbar{x}\_{i}\right)∙\left(x\_{j\_{k}}-\overbar{x}\_{j}\right)=\frac{1}{10∙(10-1)}\sum\_{}^{}\left(a\_{i}-\overbar{a}\right)∙\left(b\_{i}-\overbar{b}\right)==\frac{1}{90}\sum\_{}^{}\left(a\_{i}-33,734\right)∙\left(b\_{i}-65,606\right)=\frac{0,00296}{90}=3,289∙10^{-5}$$

$$u\_{ce}=u\_{ec}=\frac{1}{n∙(n-1)}\sum\_{}^{}\left(x\_{i\_{k}}-\overbar{x}\_{i}\right)∙\left(x\_{j\_{k}}-\overbar{x}\_{j}\right)=\frac{1}{10∙(10-1)}\sum\_{}^{}\left(a\_{i}-\overbar{a}\right)∙\left(v\_{i}-\overbar{v}\right)==\frac{1}{90}\sum\_{}^{}\left(a\_{i}-33,734\right)∙\left(v\_{i}-15,499\right)=\frac{0,00574}{90}=6,378∙10^{-5}$$

$$u\_{de}=u\_{ed}=\frac{1}{n∙(n-1)}\sum\_{}^{}\left(x\_{i\_{k}}-\overbar{x}\_{i}\right)∙\left(x\_{j\_{k}}-\overbar{x}\_{j}\right)=\frac{1}{90}\sum\_{}^{}\left(b\_{i}-65,606\right)∙\left(v\_{i}-15,499\right)=\frac{0,00226}{90}=2,511∙10^{-5}$$

$$U\_{X}=\left(\begin{matrix}u\_{x\_{1}}^{2}&u\_{x\_{1,2}}&\begin{matrix}\cdots &u\_{x\_{1,m}}\end{matrix}\\u\_{x\_{2,1}}&u\_{x\_{2}}^{2}&\begin{matrix}\cdots &u\_{x\_{2,m}}\end{matrix}\\\begin{matrix}\vdots \\u\_{x\_{m,1}}\end{matrix}&\begin{matrix}\vdots \\u\_{x\_{m,2}}\end{matrix}&\begin{matrix}\ddots &\vdots \\\cdots &u\_{x\_{m}}^{2}\end{matrix}\end{matrix}\right)=\left(\begin{matrix}u\_{cc}&u\_{cd}&u\_{ce}\\u\_{dc}&u\_{dd}&u\_{de}\\u\_{ec}&u\_{ed}&u\_{ee}\end{matrix}\right)=\left(\begin{matrix}3,338∙10^{-4}&3,289∙10^{-5}&6,378∙10^{-5}\\3,289∙10^{-5}&9,582∙10^{-4}&2,511∙10^{-5}\\6,378∙10^{-5}&2,511∙10^{-5}&7,211∙10^{-5}\end{matrix}\right)$$

$$u\_{A}^{2}=A∙U\_{x}∙A^{T}=\left(\frac{v}{2};\frac{v}{2};\frac{a+b}{2}\right)\left(\begin{matrix}u\_{cc}&u\_{cd}&u\_{ce}\\u\_{dc}&u\_{dd}&u\_{de}\\u\_{ec}&u\_{ed}&u\_{ee}\end{matrix}\right)\left(\begin{matrix}\frac{v}{2}\\\frac{v}{2}\\\frac{a+b}{2}\end{matrix}\right)=$$

$$u\_{A}^{2}=\left(\frac{\overbar{v}}{2};\frac{\overbar{v}}{2};\frac{\overbar{a}+\overbar{b}}{2}\right)\left(\begin{matrix}u\_{cc}&u\_{cd}&u\_{ce}\\u\_{dc}&u\_{dd}&u\_{de}\\u\_{ec}&u\_{ed}&u\_{ee}\end{matrix}\right)\left(\begin{matrix}\frac{\overbar{v}}{2}\\\frac{\overbar{v}}{2}\\\frac{\overbar{a}+\overbar{b}}{2}\end{matrix}\right)=$$

$$u\_{A}^{2}=\left(\frac{15,499}{2};\frac{15,499}{2};\frac{33,734+65,606}{2}\right)\left(\begin{matrix}u\_{cc}&u\_{cd}&u\_{ce}\\u\_{dc}&u\_{dd}&u\_{de}\\u\_{ec}&u\_{ed}&u\_{ee}\end{matrix}\right)\left(\begin{matrix}\frac{15,499}{2}\\\frac{15,499}{2}\\\frac{33,734+65,606}{2}\end{matrix}\right)=$$

$$u\_{A}^{2}=\left(7,7495;7,7495;49,67\right)∙\left(\begin{matrix}3,338∙10^{-4}&3,289∙10^{-5}&6,378∙10^{-5}\\3,289∙10^{-5}&9,582∙10^{-4}&2,511∙10^{-5}\\6,378∙10^{-5}&2,511∙10^{-5}&7,211∙10^{-5}\end{matrix}\right)∙\left(\begin{matrix}7,7495\\7,7495\\49,67\end{matrix}\right)=$$

$$u\_{A}^{2}=\left(0,006009;0,008928;0,004271\right)∙\left(\begin{matrix}7,7495\\7,7495\\49,67\end{matrix}\right)=0,327877 mm^{4}$$

$$u\_{c}=\sqrt{u\_{A}^{2}+u\_{B}^{2}}=\sqrt{0,327877+0}=0,572605449 mm^{2}$$

$$U=k\_{p}∙u\_{c}=2∙0,572605449=1,145211 mm^{2}$$

Výsledok:

$$S=\left(769,8\pm 1,2\right) mm^{2}$$